AD-A047 531 AERONAUTICAL RESEARCH LABS MELBOURNE (AUSTRALIA) F/G 1/3
THE EFFECT OF SPANWISE GUST VARIATIONS ON THE TRANSFER FUNCTION--ETC(U)
AUG 76 D J SHERMAN
UNCLASSIFIED ARL/STRUC NOTE-431 NL

1 OF 1
ADA047531

END
DATE
FILMED
1 - 78
DDC

ARL/Struc. Note 431

AR No. 228

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE VICTORIA

See last page

Structures Note 431

THE EFFECT OF SPANWISE GUST VARIATIONS ON THE TRANSFER FUNCTION OF AN AIRCRAFT MODEL WITH ONE DEGREE OF FREEDOM

DOUGLAS JOHN SHERMAN

AUGUST 1976

136

AR No. 228

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

STRUCTURES NOTE 431

THE EFFECT OF SPANWISE GUST VARIATIONS ON THE TRANSFER FUNCTION OF AN AIRCRAFT MODEL WITH ONE DEGREE OF FREEDOM

by

DOUGLAS JOHN SHERMAN

SUMMARY

Charts are derived for the determination of the power spectrum parameters A *and* N_0 SUB 0 *for vertical acceleration, at an aircraft's centre of gravity, due to atmospheric turbulence. The method takes account of spanwise variations in gust loading, and so overcomes the paradox of an infinite* N_0 *which is found with a one-dimensional gust model.*

POSTAL ADDRESS: Chief Superintendent, Aeronautical Research Laboratories, Box 4331, P.O., Melbourne, Victoria, 3001, Australia.

CONTENTS

NOTATION

A	σ_y/σ_w
A	Coefficient
a	Reference time
$\mathcal{R}$	Aspect ratio
B	Coefficient
C_L	Lift coefficient
$C(\omega)$	Theodorsen function
c	Wing chord
D	Coefficient
E	Coefficient
$F(\omega)$	Factor to allow for transverse variation in wing loading
$\mathscr{F}[\ ]$	Operator denoting Fourier transform
f	Frequency c/s
f	Interpolation fraction
$g(y)$	Wing loading at transverse section y
H	Transfer function
H_1	Transfer function for 1-D gust field
H_2	Transfer function for 2-D gust field
$H_n^{(2)}(\omega)$	Hankel function
$J_n(\omega)$	Bessel function
K_Φ/μ	Non-dimensional measure of A
k_0	Non-dimensional measure of N_0
L	Integral scale of turbulence
N_0	Rate of level crossings of mean value
P_w	Power spectral density of w
P_y	Power spectral density of y
S	Wing area
$S(\omega)$	Sears function
s	Distance travelled in time t
t	Time
U	Longitudinal velocity component of aircraft
W	Aircraft weight
W_g	$\mathscr{F}[w_g]$
w_g	Vertical component of gust velocity
$Y_n(\omega)$	Bessel function

y	General response of aircraft, and in particular y is often taken to be the vertical acceleration at the aircraft centre of gravity
y	Transverse co-ordinate
Z	$\mathcal{F}(z)$
z	Elevation of aircraft
α	Angle of attack
$\dot{\alpha}$	Coefficient
β	Coefficient
γ	Coefficient
δ	Coefficient
ϵ	Coefficient
μ	Aircraft mass parameter
ρ	Air density
σ	Distance travelled in time τ
σ_w	Standard deviation of w
σ_y	Standard deviation of y
τ	Time
$\Phi(s)$	Wagner function
$\Psi(s)$	Küssner function
ω	Non-dimensional frequency
ω_c	Cut-off frequency

Note: A dot denotes differentiation w.r.t. time

1. INTRODUCTION

The power spectral method of analysing gust loads on an aircraft (see ref. 1) requires the evaluation of two parameters, A and N_0. If $y(t)$ is the output of some transducer (in the present case we consider the vertical acceleration measured at the aircraft's centre of gravity), $w(t)$ is the vertical gust velocity, $P_w(f)$ is the power spectrum of the gust velocity and $P_y(f)$ is the power spectrum of $y(t)$, we define

$$A = \frac{\sigma_y}{\sigma_w} \approx \frac{1}{\sigma_w}\left[\int_0^\infty P_y(f)df\right]^{1/2} \tag{1.1}$$

$$N_0 = \frac{\sigma_y'}{\sigma_y} \approx \left[\frac{\int_0^\infty f^2 P_y(f)df}{\int_0^\infty P_y(f)df}\right]^{1/2} \tag{1.2}$$

where $P_w(f)$ is assumed to follow the von Karman expression:

$$P_w(f) = \frac{\sigma_w{}^2}{\pi} \times \frac{2\pi L}{U} \times \frac{1+\frac{8}{3}(af)^2}{[1+(af)^2]^{11/6}} \tag{1.3}$$

$$a = 1 \cdot 339 \times 2\pi \times \frac{L}{U} \tag{1.4}$$

where

U is the aircraft forward velocity and
L is the integral scale of turbulence;

asymptotically $P_w(f)$ behaves as $f^{-5/3}$.

The transfer function of the aircraft is

$$H(f) = \mathscr{F}[y(t)]/\mathscr{F}[w(t)] \tag{1.5}$$

where $\mathscr{F}[\ldots]$ denotes the Fourier transform of the quantity in brackets. We have

$$P_y(f) = |H(f)|^2 P_w(f) \tag{1.6}$$

The transfer function may be derived by obtaining the response of the aircraft to a sinusoidal gust of arbitrary frequency. For the simple case of an aircraft with one degree of freedom (heave), and a one dimensional variation of gust (gust velocity varies sinusoidally along the flight path but is invariant in the spanwise or vertical direction) the transfer function is well known. It is given (see Appendix) by

$$H_1(f) = \frac{2\frac{U}{c}i\omega S(\omega)}{(2\mu+\frac{1}{2})i\omega+C(\omega)} \tag{1.7}$$

where the Sears function, $S(\omega)$, is a complex expression which represents the unsteady wake effect caused by gust, and the Theodorsen function, $C(\omega)$, is another complex expression representing the unsteady wake effect caused by a change in the angle of attack. (Note that a vertical velocity of the aircraft produces an aerodynamic effect equivalent to a change in angle of attack.) The parameter ω is the reduced frequency

$$\omega = \frac{2\pi f(c/2)}{U} \tag{1.8}$$

measured in radians per semi-chord, and μ is the mass parameter

$$\mu = \frac{2(W/S)}{\rho c g(dC_L/d\alpha)} \tag{1.9}$$

The transfer function (1.7), in combination with, say, the von Karman power spectrum leads to an infinite value for N_0. This is basically because of the limitations of the one-dimensional gust model, as the high frequencies which contribute largely to the numerator of equation 1.2 are not realistically modelled by a gust which is invariant across the span. In the next section we consider a gust structure which varies sinusoidally in the spanwise direction as well as the axial direction. At high wave numbers (isotropic turbulence is simulated by making the wave numbers in the axial and spanwise directions equal) the upgusts and downgusts on the wing cancel and a realistic value is obtained for N_0.

2. THE EFFECT OF SPANWISE VARIATIONS IN THE GUST STRUCTURE

Consider a gust which varies sinusoidally across the span according to the equation

$$w = w_g \cos \frac{2\pi f y}{U} \tag{2.1}$$

(This equation involves the simplification that the gust is symmetrical across the span, but this simplification is without loss of generality as rolling motions caused by an asymmetry would not affect the vertical acceleration at the centre of gravity.)

Using the "strip theory" assumption that each chordwise element behaves as a two-dimensional aerofoil, we obtain the total lift on the wing as the lift corresponding to the updraft at the centre-section, multiplied by the factor

$$F(\omega) = \frac{1}{\text{span}} \int_{-\text{span}/2}^{\text{span}/2} g(y) \cos \frac{2\pi f y}{U} dy \tag{2.2}$$

where $g(y)$ is the wing loading at the section y. The precise form of the function $g(y)$ is only of secondary importance, and we have made the simple assumption that the wing loading at any section is proportional to the chord at that section. We have also assumed that the wing plan form varies from a delta wing at aspect ratios of two or less, through trapezoidal to rectangular at aspect ratios of eight or greater. The formula for the chord of a trapezoidal wing is

$$\text{chord} = \text{mean chord} \left[1 + \gamma\left(\frac{1}{4} - \frac{|y|}{\text{span}}\right)\right] \tag{2.3}$$

where y is the spanwise co-ordinate measured from the centre of the fuselage, and γ varies from 0 at an aspect ratio of 8 or larger, to 4 (corresponding to a delta wing) at an aspect ratio of 2 or less, according to the formula

$$\left.\begin{aligned} \gamma &= 4\left[\frac{8 - A\!\!R}{6}\right] \\ 2 &< A\!\!R < 8 \end{aligned}\right\} \tag{2.4}$$

Accordingly the factor in equation 2.2 becomes

$$F(\omega) = \frac{1}{\text{span}} \int_{-\text{span}/2}^{\text{span}/2} \left[1 + \gamma\left(\frac{1}{4} - \frac{|y|}{\text{span}}\right)\right] \quad \frac{2\pi f y}{U} dy \tag{2.5}$$

$$= \left(1 - \frac{\gamma}{4}\right) \frac{\sin \omega A\!\!R}{\omega A\!\!R} + \frac{\gamma}{4}\left[\frac{\sin(\omega A\!\!R/2)}{(\omega A\!\!R/2)}\right]^2 \tag{2.6}$$

and so for trapezoidal or rectangular wings ($\gamma \neq 4$), $|F(\omega)|^2$ behaves asymptotically as ω^{-2}, whilst for delta wings ($\gamma = 4$), $|F(w)|^2$ behaves asymptotically as ω^{-4}.

Still using the "strip theory", we can now write the transfer function for two-dimensional gusts as

$$|H_2(\omega)|^2 = |H_1(\omega)|^2 \,.\, |F(\omega)|^2 \tag{2.7}$$

$$= |H_0(\omega)|^2 \,.\, |S(\omega)|^2 \; |F(\omega)|^2 \tag{2.8}$$

Figures 3 and 4 show graphs of the transfer functions $|H_0(f)|^2$, $|H_1(f)|^2$, and $|H_2(f)|^2$ together with input and output spectra. Figure 4 differs from Figure 3 only in the formulae used to evaluate the effects of the non-steady aerodynamics. Figure 3 is based on the exact equations A.20 and A.21 (see Appendix), whereas Figure 4 is based on the fitted curves, A.20*a* and A.21*a*, which, it may be seen, depart from the former equations at non-dimensional frequencies above 10.

3. THE EVALUATION OF A AND N_0

The integrals which arise in a numerical evaluation of A and N_0 are

$$M_0 = \int_0^{f_c} |H(f)|^2 P_w(f) df \tag{3.1}$$

$$M_2 = \int_0^{f_c} f^2 |H(f)|^2 P_w(f) df \tag{3.2}$$

and the first of these will converge to a limit with increasing f_c because the integrand converges much faster than f^{-1}. However the integrand in the second integral will only give rise to a convergent expression if the transfer function corresponding to a two-dimensional gust structure is used; the simple one-dimensional gust model gives rise to a non-convergent (infinite) integral. The fact has sometimes led to an implicit assumption that N_0 is theoretically infinite, and that some arbitrary procedure is necessary to obtain a realistic finite value of N_0. Recently Houbolt (ref. 2) has given a method for evaluating N_0 based on a two-dimensional gust structure model, but earlier work (Houbolt (refs. 3, 4)) in which a design manual for vertical gusts had been given, was based on a one-dimensional gust model. There, it was recommended that a cut-off frequency, f_c, be chosen such that equation 3.1 was within a small fraction (2%) of its limit value, and that the same cut-off frequency be applied to equation 3.2. It so happens that this procedure gives approximately the right answer for the most common cases with an aspect ratio around 4 to 8 and the mass parameter, μ, in the range 10 to 50. Houbolt remarks that the N_0 value derived from the model should be increased by a factor of the order of 2 to allow for flexibility effects, and similarly the A value should be increased by about 10%. Houbolt states "this factor may be adjusted upward or downward if some additional insight relative to the aeroplane response characteristics is on hand". The method proposed here takes account of aspect ratio effects in a definite way which would otherwise be merely left to the engineer's judgement.

The transfer function of the aircraft vertical acceleration from two dimensional vertical gusts is a function of the aspect ratio, $\mathcal{R}$, and the non-dimensional frequency, ω, where

$$\omega = \frac{2\pi f(c/2)}{U} \tag{3.3}$$

However, the von Karman power spectrum equation is a function of a different form of non-dimensional frequency, af, where

$$af = \frac{2\pi f \, 1 \cdot 339 L}{U} = 1 \cdot 339 \times \frac{2L}{c} \times \omega \tag{3.4}$$

and so the solutions to equations 1.1 and 1.2 will have, as parameter, $2L/c$. Using equations 2.7, 2.6 and A.29 we may conveniently obtain the solutions in the non-dimensional form

$$A = \frac{U}{cg}\frac{K_\Phi}{\mu} \tag{3.5}$$

$$N_0 = \frac{U}{\pi c} k_0 \tag{3.6}$$

where

$$\left(\frac{K_\Phi}{\mu}\right)^2 = \int_0^{\omega c} \frac{|F(\omega)|^2 \, |S(\omega)|^2}{|(2\mu+\frac{1}{2})i\omega+C(\omega)|^2} \frac{(2\omega)^2}{\pi} \frac{2L}{c} \frac{1+\frac{8}{3}\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2}{\left[1+\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2\right]^{11/6}} \, d\omega \tag{3.7}$$

$$k_0{}^2 = \frac{\displaystyle\int_0^{\omega c} \frac{|F(\omega)|^2 \, |S(\omega)|^2 \, (2\omega)^2}{|(2\mu+\frac{1}{2})i\omega+C(\omega)|^2} \frac{1+\frac{8}{3}\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2}{\left[1+\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2\right]^{11/6}} \, \omega^2 \, d\omega}{\displaystyle\int_0^{\omega c} \frac{|F(\omega)|^2 \, |S(\omega)|^2 \, (2\omega)^2}{|(2\mu+\frac{1}{2})i\omega+C(\omega)|^2} \frac{1+\frac{8}{3}\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2}{\left[1+\left(1\cdot339\,\frac{2L}{c}\,\omega\right)^2\right]^{11/6}} \, d\omega} \tag{3.8}$$

The convergence of K_Φ/μ and k_0 towards a limit as the cut-off non-dimensional frequency, ω_c, is raised as shown in Figure 5. It may be seen that A (or K_Φ/μ) attains its limit (or nearly attains its limit) at a slightly lower cut-off frequency than does N_0 (or k_0).

DISCUSSION

Using equations 3.7 and 3.8 charts have been derived for the determination of A (see Fig. 6) and N_0 (see Fig. 7). Houbolt's chart for A is, as would be expected, very similar to the charts presented here for the low aspect ratio case. There is more variation in the charts for N_0, and Figure 8 shows how both A and N_0 are affected by variations in aspect ratio. A is little affected except at large aspect ratio and low values of μ. However N_0 is strongly affected by aspect ratio, and this variation seems to be nearly independent of μ and $2L/c$ except at quite large aspect ratio.

The method proposed here is most likely to be in error at low aspect ratios where the "strip theory" is at best uncertain, but the following example (Table 1) shows that even for an aspect ratio as low as 2 the method produces reasonable agreement with observation.

TABLE 1

Basic Data	
Aircraft	Mirage IIIO
Altitude	600 m
Speed	232 m/s
Aspect ratio	1·97
Aerodynamic mean chord	5·25 m
μ	32
$2L/c$	88
K_Φ/μ	0·022
A	$0 \cdot 1 \dfrac{g}{\text{m/sec}}$
Determination of N_0 *by various Methods*	
(a) One-dimensional gust model (as modified by Houbolt, refs. 3 to 5)	$k_0 = 0 \cdot 073$ $N_0 = 1$ Hz
(b) Two-dimensional gust model (Fig. 7)	$k_0 = 0 \cdot 13$ $N_0 = 1 \cdot 8$ Hz
(c) Experimental—Rigid aircraft	$N_0 = 2$ Hz
(d) Experimental—Flexible aircraft	$N_0 = 3 \cdot 4$ Hz

For this example, the experimental determination of N_0 has been described by Sherman (ref. 6). In brief the power spectrum was calculated out to a Nyquist frequency of 30 Hz, using a 100 second record of vertical acceleration during fairly severe turbulence. (Part of flight 2-135 as described by Rider *et al.* (ref. 7).) The principal structural flexibility effect was due to a fundamental wing bending mode at about 11 Hz. The N_0 value was determined from equation 1.2 modified to have a finite cut-off frequency. For the flexible aircraft N_0 was determined by integrating right up to 30 Hz; whilst an estimate of N_0 for a rigid aircraft was obtained by reducing the cut-off frequency to 9 Hz, which eliminated most of the fundamental structural resonance mode. The rigid (one-dimensional gust) model of Houbolt estimates an N_0 of only 1 Hz, whereas the rigid (two-dimensional gust) model on which Figure 7 is based estimates an N_0 of 1·8 Hz, which compares well with the experimental determination of 2 Hz for a rigid aircraft. The effect of structural flexibility increased N_0 to 3·4 Hz, which agrees well with Houbolt's suggested 100% increase to allow for structural flexibility.

This one set of results does not of itself validate the strip theory at low aspect ratios, but it does provide support for the method proposed herein for estimating N_0.

5. CONCLUSION

The charts for determination of A and N_0 presented here appear to give more realistic values of N_0 than some of the other charts presently in use.

REFERENCES

1.	U.S.A.F.	Military specification—Airplane strength and rigidity (Flight loads) MIL A 008861A (U.S.A.F.), 31 March 1971.
2.	Houbolt, J. C.	Effect of non-uniform spanwise gusts on aircraft vertical response. U.S.A.F.—AFOSR-TR-74-0754, January 1974.
3.	Houbolt, J. C.	Design manual for vertical gusts based on power spectral techniques. U.S.A.F.—AFFDL-TR-70-106, 1970.
4.	Houbolt, J. C.	Frequency response functions and human pilot modelling. N.A.T.O. —AGARD-R-580-71, March 1971.
5.	Houbolt, J. C.	Recommended procedures for processing acceleration data obtained by aircraft during atmospheric trubulence encounter. AGARD R-631, July 1975.
6.	Sherman, D. J.	The cumulative exceedance distribution for accelerations due to turbulence encountered by a Mirage. A.R.L. Tech. Memo SM228, February 1975.
7.	Rider, C. K., Thomson, M. R., and Verinder, F. E.	Measurement of extreme mechanical turbulence during low level flights by Mirage A3-76. A.R.L. Report SM333, May 1971.
8.	Jones, R. T.	The unsteady lift of a wing of finite aspect ratio. N.A.C.A. Report 681, 1940
9.	Zbrozek, J. K.	Gust alleviation factor. Aeronautical Research Council, R. & M. 2970, May 1953.
10.	Bisplinghoff, Ashley, and Halfman	Aeroelasticity. Addison Wesley, 1955.
11.	Sears, W. R.	Some aspects of non-stationary airfoil theory and its practical application. Journal of the Aeronautical Sciences, vol. 8, no. 3, January 1941.

APPENDIX

The Transfer Function for 1-D Gusts

Consider a rigid aircraft, constrained against rotation or lateral motion, flying at a constant speed, U, and altitude, z, with a lift force $L = \frac{1}{2}\rho U^2 S C_L$. If the aircraft now encounters a vertical gust velocity, w_g, and rises with a vertical velocity, $\dot{z}$, the pseudo-steady (i.e. if dynamic effects were to be neglected) increment in lift would be

$$\Delta L = \tfrac{1}{2}\rho U^2 S \frac{dC_L}{d\alpha}\left[\frac{w_g - \dot{z}}{U}\right] \tag{A.1}$$

$$= \text{Lift due to gust} - \text{Lift due to vertical motion}$$

The two components in this equation are both modified by the dynamic response of the system, but each in a different way because the change in incidence due to gust is felt first at the leading edge of the wing, and progresses across the wing at speed U, whereas the change in incidence due to aircraft vertical motion affects the whole wing simultaneously.

When an aerofoil changes its incidence by an amount $\delta\alpha$ at time $t = 0$, the extra lift δC_L builds up gradually to its final value, $\delta C_{L\infty}$. We denote by s the (non-dimensional) distance travelled since the time $t = 0$, in units of the semi-chord, so that

$$s = \frac{Ut}{c/2} \tag{A.2}$$

We have

$$\delta C_L = \delta C_{L\infty} \,.\, \Phi(s) \tag{A.3}$$

where $\Phi(s)$ is known as the Wagner function in the case of the two-dimensional wing, and, according to Jones (ref. 8) may be approximated in the three-dimensional case by the formula

$$\left.\begin{aligned} s < 0:&\quad \Phi(s) = 0 \\ s > 0:&\quad \Phi(s) = 1 - D \,.\, \exp(-\delta s) - E \,.\, \exp(-\epsilon s) \end{aligned}\right\} \tag{A.4}$$

where D. δ, E and ϵ are functions of aspect ratio.

Similarly when an aerofoil penetrates a unit step gust at $t = 0$, the lift builds up as

$$\delta C_L = \delta C_{L\infty} \Psi(s) \tag{A.5}$$

where $\Psi(s)$ is known as the Küssner function in the case of the two-dimensional wing and may be approximated in the three-dimensional (finite span) case by the formula

$$\left.\begin{aligned} s < 0:&\quad \Psi(s) = 0 \\ s > 0:&\quad \Psi(s) = 1 - A \,.\, \exp(-\alpha s) - B \,.\, \exp(-\beta s) \end{aligned}\right\} \tag{A.6}$$

where A, α, B and β are functions of aspect ratio, as in the following table (see Zbrozek (ref. 9) and Bisplinghoff *et al.* (ref. 10)):

Aspect Ratio	Küssner function $\Psi(s)$				Wagner function $\Phi(s)$			
	A	α	B	β	D	δ	E	ϵ
∞	0·5	0·13	0·5	1·0	0·335	0·3	0·165	0·0455
6	0·48	0·294	0·334	0·965	0·361	0·381	0	
3	0·679	0·558	0·227	3·20	0·283	0·54	0	

For calculation purposes we need an interpolation formula to give the value of Φ or Ψ at other aspect ratios than the ones given in the table. If $AR1$ and $AR2$ are two of the aspect ratios for which the Φ and Ψ functions are known, and AR is some intermediate aspect ratio with $AR1 < AR < AR2$, then the interpolation formula used here is

$$\left.\begin{aligned} f &= \frac{\sqrt{1/AR} - \sqrt{1/AR2}}{\sqrt{1/AR1} - \sqrt{1/AR2}} \\ \Phi_{AR} &= f\,\Phi_{AR1} + (1-f)\Phi_{AR2} \\ \Psi_{AR} &= f\,\Psi_{AR1} + (1-f)\Psi_{AR2} \end{aligned}\right\} \tag{A.7}$$

To extrapolate the values of Φ and Ψ for aspect ratios below 3 (it should be noted that a strip theory such as we will use later is not an accurate approximation at such low aspect ratios) we have extended the above table by assuming that for aspect ratios of 0·1 or less the values of A, B, D and E attain their limiting values of zero.

The theory on which the expressions for $\Phi(s)$ and $\Psi(s)$ have been derived is a linear one, so the effect of complex motions can be built up by a process of superposition. The result, as we now show, is a convolution (also called Duhamel or superposition) integral. If the vertical aircraft velocity changes from $\dot{z}$ to $\dot{z}+\delta\dot{z}$ whilst the aircraft position changes from $s = \sigma$ to $s = \sigma+\delta\sigma$, the increment in lift at some later position, s (see Fig. 2), is

$$\delta C_L = \delta C_{L\infty}\Phi(s-\sigma) = \frac{dC_L}{d\alpha}\frac{\delta\dot{z}}{U}\Phi(s-\sigma)$$

So the total increment in lift since time $t = 0$ is

$$\Delta C_L = \int_0^s \frac{dC_L}{d\alpha}\frac{d\dot{z}}{d\sigma}\frac{1}{U}\Phi(s-\sigma)d\sigma \tag{A.8}$$

and with

$$s = \frac{Ut}{c/2} \qquad \sigma = \frac{U\tau}{c/2} \tag{A.9}$$

we obtain

$$\Delta C_L = \frac{1}{U}\frac{dC_L}{d\alpha}\int_0^t \frac{d\dot{z}(\tau)}{d\tau}\Phi\left[\frac{2U}{c}(t-\tau)\right]d\tau \tag{A.10}$$

and similarly for changes δw_g in the vertical gust velocity we obtain

$$\Delta C_L = \frac{1}{U}\frac{dC_L}{d\alpha}\int_0^t \frac{dw_g(\tau)}{d\tau}\Psi\left[\frac{2U}{c}(t-\tau)\right]d\tau \tag{A.11}$$

The convolution integrals (equation A.10 and A.11) can conveniently be solved by Fourier transformation, and for this purpose we will need the Fourier transforms (possibly requiring to be understood in the sense of generalised functions) of the functions Φ and Ψ. To this end, consider the case of the harmonic oscillations

$$\dot{z} = \dot{z}_0 \exp(2\pi i f \tau) \tag{A.12}$$

or

$$w_g = w_0 \exp(2\pi i f \tau) \tag{A.13}$$

The solutions to these two cases have been given by Sears (ref. 11) as

$$\Delta C_L = \frac{\dot{z}_0}{U} 2\pi[C(\omega)+\tfrac{1}{2}i\omega] \exp(2\pi i f t) \tag{A.14}$$

and

$$\Delta C_L = \frac{w_0}{U} 2\pi S(\omega) \exp(2\pi i f t) \tag{A.15}$$

respective, where

$$S(\omega) = [J_0(\omega)-iJ_1(\omega)]C(\omega)+iJ_1(\omega) \tag{A.16}$$

$$C(\omega) = \frac{H_1^{(2)}(\omega)}{H_1^{(2)}(\omega)+iH_0^{(2)}(\omega)} \tag{A.17}$$

$$H_n^{(2)}(\omega) = J_n(\omega)-iY_n(\omega) \tag{A.18}$$

in which $S(\omega)$ has become known as the Sears function, and $C(\omega)$ is the Theodorsen function. $H_n^{(2)}(\omega)$ is the Hankel function of the second kind, and ω is the non-dimensional frequency based on the wing semi-chord,

$$\omega = \frac{2\pi f(c/2)}{U} \tag{A.19}$$

Sears considered a Joukowski aerofoil for which $dC_L/d\alpha = 2\pi$. With this substitution we may equate ΔC_L from equation 10 or 11 with the value of ΔC_L in equation 14 or 15. Then using equations 12 and 13 and taking the (formal) Fourier transform we obtain

$$\mathscr{F}[\Phi] = \frac{C(\omega)+\tfrac{1}{2}i\omega}{2\pi i f} \tag{A.20}$$

$$\mathscr{F}[\Psi] = \frac{S(\omega)}{2\pi i f} \tag{A.21}$$

The first of these equations differs a little from the result usually quoted, in the inclusion of the term $\tfrac{1}{2}i\omega$, which includes the effect of virtual mass (the so-called non-circulating lift].

As an alternative to equations A.20 and A.21 we may use the numerical approximations in equations A.4 and A.6 to obtain

$$\mathscr{F}[\Phi] = \int_0^\infty \left[1 - D\exp\left(-\delta\frac{2U}{c}t\right) - E\exp\left(-\epsilon\frac{2U}{c}t\right)\right]. \exp(-2\pi i f t)\, dt$$

$$= \frac{1}{2\pi i f}\left\{1 - \frac{D}{1+\dfrac{\delta}{i\omega}} - \frac{E}{1+\dfrac{\epsilon}{i\omega}}\right\} \tag{A.20a}$$

$$\mathscr{F}[\Psi] = \frac{1}{2\pi i f}\left\{1 - \frac{A}{1+\dfrac{\alpha}{i\omega}} - \frac{B}{1+\dfrac{\beta}{i\omega}}\right\} \tag{A.21a}$$

However, using these equations it will be necessary to account separately for the effect of virtual mass.

Equation A.1 can now be modified to take account of dynamic effects:

$\Delta L =$ Lift due to gusts—Lift due to a/c vertical motion

$$= \tfrac{1}{2}\rho U^2 S \frac{dC_L}{d\alpha}\left\{\frac{1}{U}\int_0^t \frac{dw_g}{d\tau}\Psi\left[\frac{2U}{c}(t-\tau)\right]d\tau - \frac{1}{U}\int_0^t \frac{d\dot{z}(\tau)}{d\tau}\Phi\left[\frac{2U}{c}(t-\tau)\right]d\tau\right\} \tag{A.22}$$

and the vertical equation of motion may be written

$$\Delta L = \frac{W}{g} z \tag{A.23}$$

which with

$$\mu = \frac{2W/S}{\rho c g dC_L/d\alpha} \tag{A.24}$$

becomes

$$\frac{\mu c}{U}\ddot{z} + \int_0^t \frac{d\dot{z}(\tau)}{d\tau}\Phi\left[\frac{2U}{c}(t-\tau)\right]d\tau = \int_0^t \frac{dw_g(\tau)}{d\tau}\Psi\left[\frac{2U}{c}(t-\tau)\right]d\tau \tag{A.25}$$

Denoting by Z and W_g the Fourier transforms of z and w_g respectively, this equation becomes, under Fourier transformation

$$\left\{\frac{\mu c}{U}(2\pi i f)^2 + (2\pi i f)^2 \mathscr{F}[\Phi]\right\}Z = 2\pi i f\, W_g \mathscr{F}[\Psi] \tag{A.26}$$

and since the Fourier transform of the aircraft acceleration is

$$\mathscr{F}[z] = (2\pi i f)^2 Z$$

the transfer function of the aircraft becomes, for two-dimensional gusts,

$$H_1(f) = \frac{\mathscr{F}(\ddot{z})}{W_g} = \frac{(2\pi i f)\mathscr{F}[\Psi]}{\dfrac{\mu c}{U} + \mathscr{F}[\Phi]} \tag{A.27}$$

$$= \frac{\left(\dfrac{2i\omega U}{c}\right)(2\pi i f)\mathscr{F}[\Psi]}{2\mu\omega i + (2\pi i f)\mathscr{F}[\Phi]} \tag{A.28}$$

On substituting the expressions A.20 and A.21 we obtain

$$|H_1(f)|^2 = \frac{\left(\dfrac{U}{c}\right)^2 (2\omega)^2 |S(\omega)|^2}{|(2\mu+\tfrac{1}{2})i\omega + C(\omega)|^2} \tag{A.29}$$

$$= |H_0(f)|^2 |S(\omega)|^2 \tag{A.30}$$

where

$$H_0(f) = \frac{\left(\frac{U}{c}\right)(2i\omega)}{(2\mu+\frac{1}{2})i\omega + C(\omega)} \tag{A.31}$$

is the transfer function obtained by neglecting the effect of unsteady aerodynamics of gust penetration (but not of the less important effect of unsteady aerodynamics of the vertical motion of the aircraft).

It may be seen that the effect of the virtual mass is, as would be expected, the same as a slight increase in the mass parameter, μ, equivalent to increasing the aircraft mass by the mass of a cylinder of air of diameter equal to the wing chord.

The function $C(\omega)$ behaves asymptotically as a non-zero constant, $|S(\omega)|^2$ behaves as $1/(1+2\pi\omega)$ (the Liepman approximation) and so $|H_0(f)|^2$ behaves asymptotically as ω^{-1}.

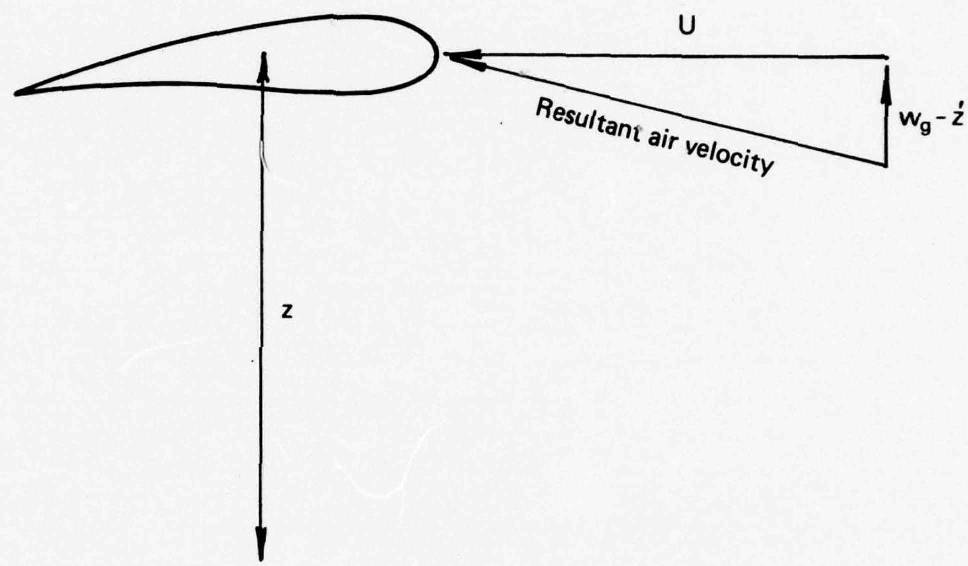


FIG. 1 CHANGE IN ANGLE OF ATTACK DUE TO GUST VELOCITY
AND AIRCRAFT VERTICAL MOTION

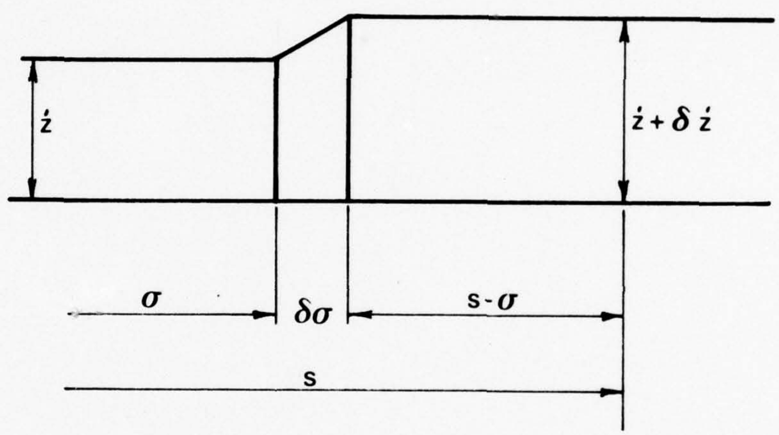


FIG. 2 DEVELOPMENT OF CONVOLUTION INTEGRAL

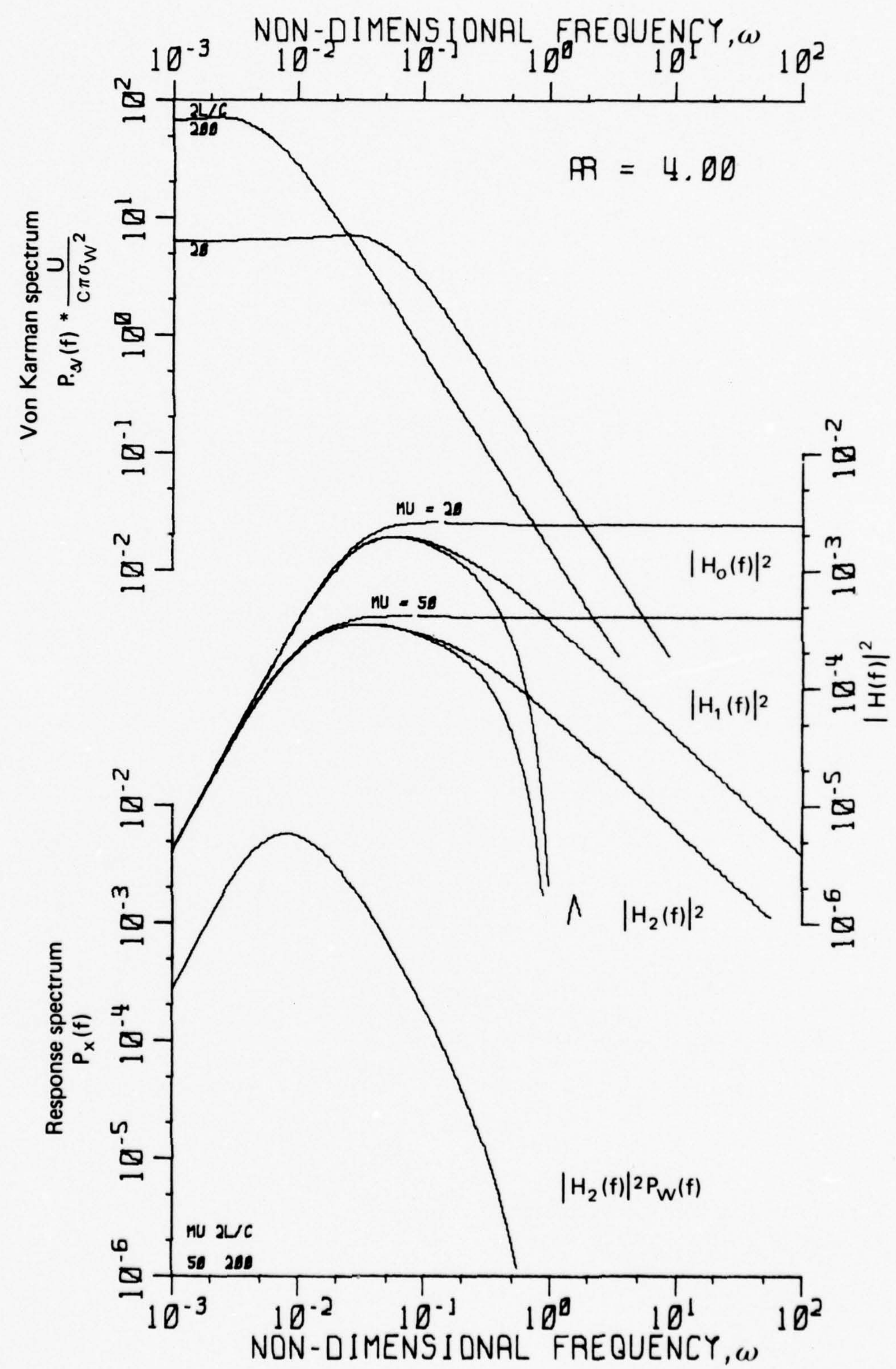


FIG. 3 INPUT SPECTRUM, TRANSFER FUNCTION AND OUTPUT (RESPONSE) SPECTRUM (USING EQUATIONS A.20, A.21)

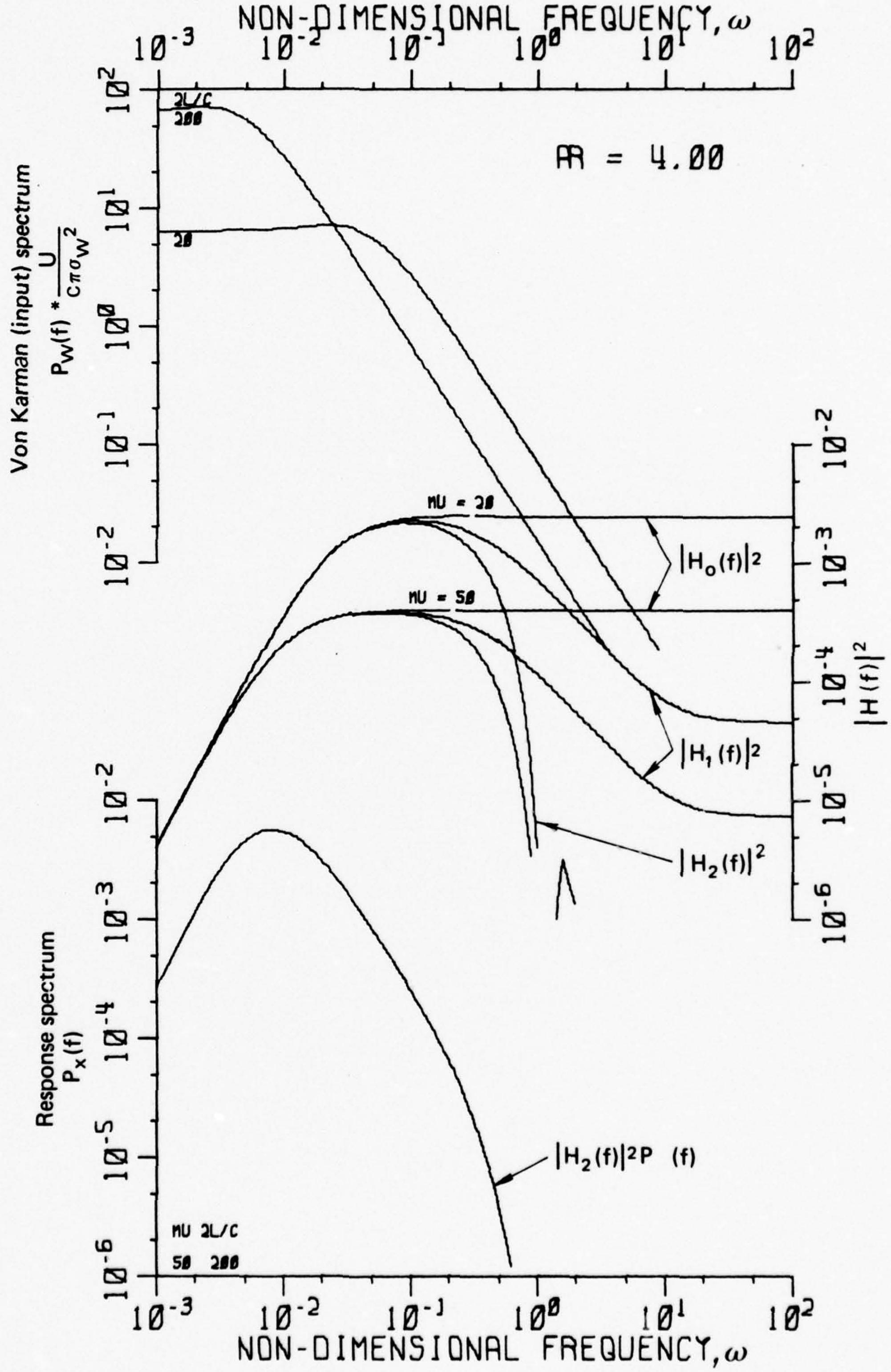
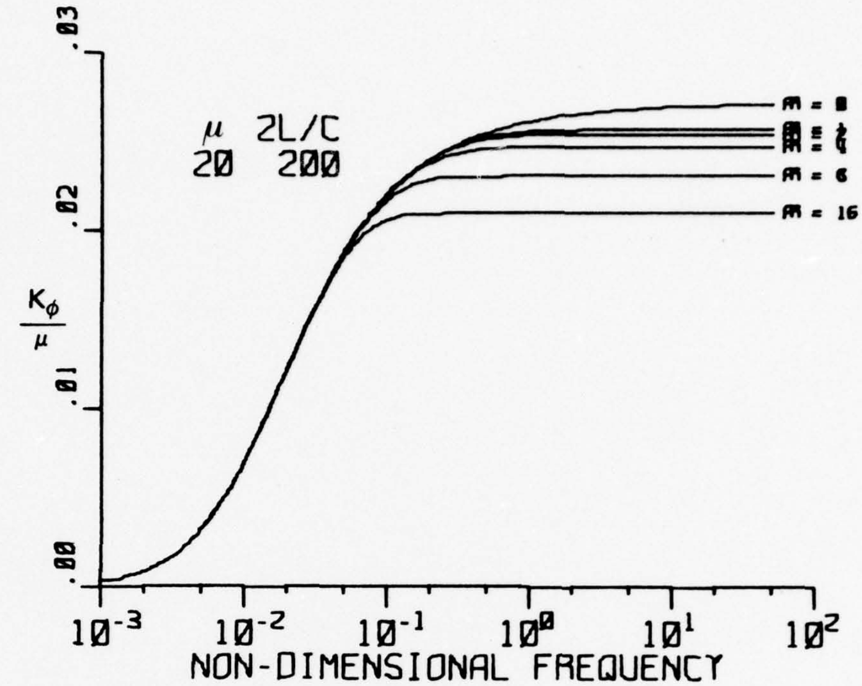


FIG. 4 INPUT SPECTRUM, TRANSFER FUNCTION AND OUTPUT (RESPONSE) SPECTRUM (USING APPROXIMATE EQUATIONS A.20a, A.21a)



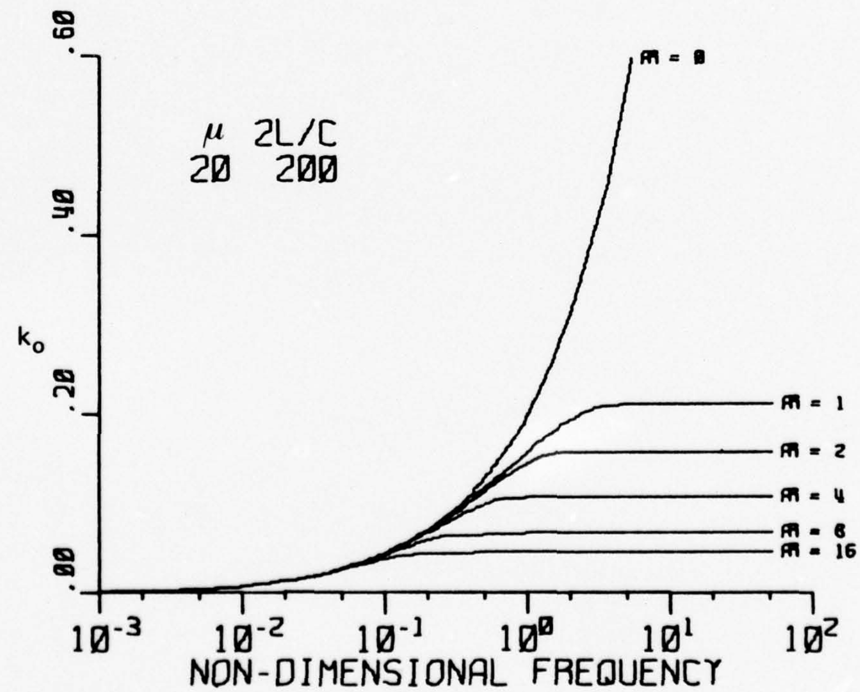


FIG. 5 CONVERGENCE OF THE INTEGRALS FOR $\frac{K_\phi}{\mu}$ AND k_o TOWARDS A LIMIT AS UPPER BOUND OF INTEGRAL BECOMES LARGE

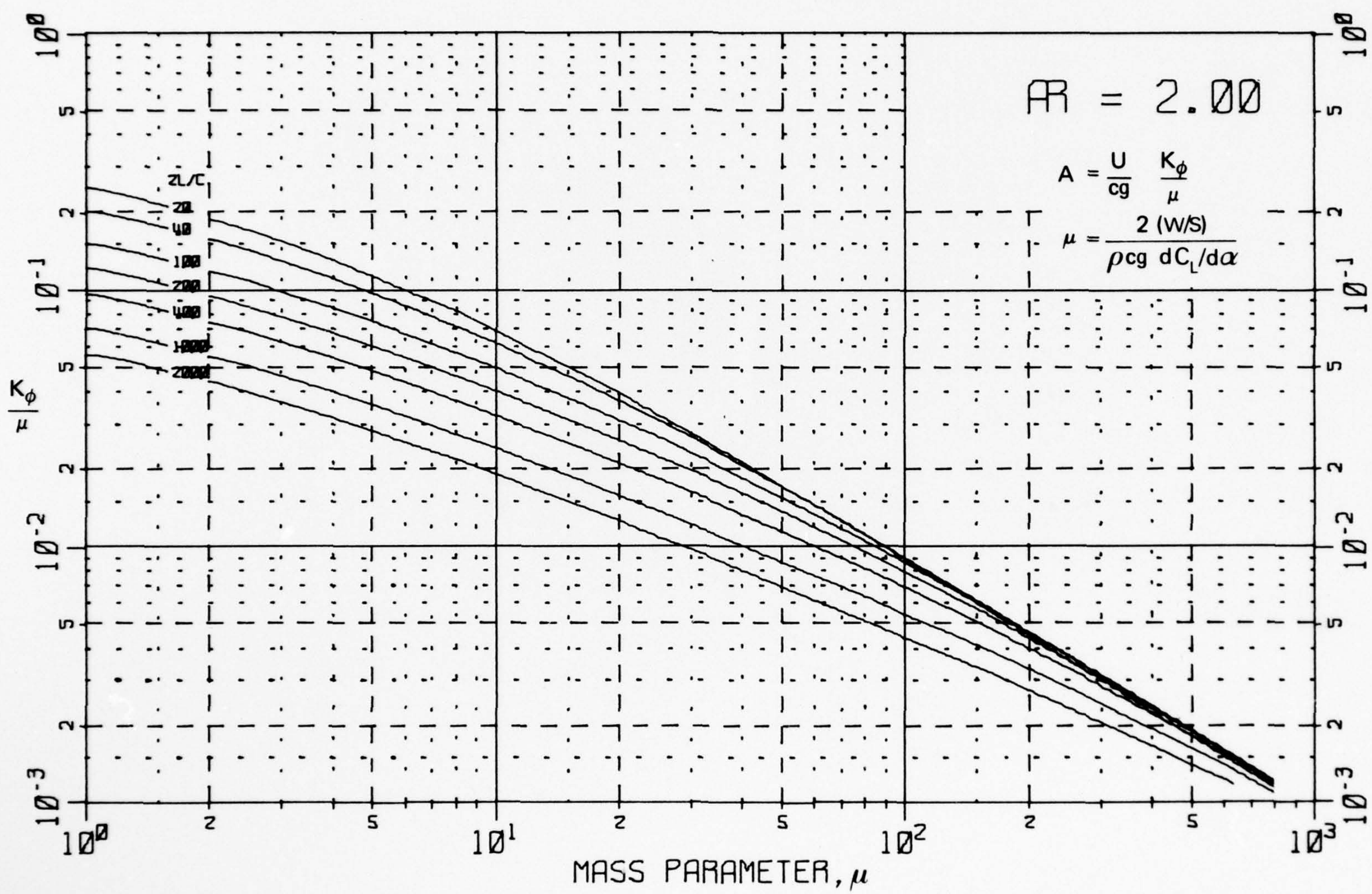


FIG. 6 CHARTS FOR THE DETERMINATION OF N_0 FOR c.g. VERTICAL ACCELERATION

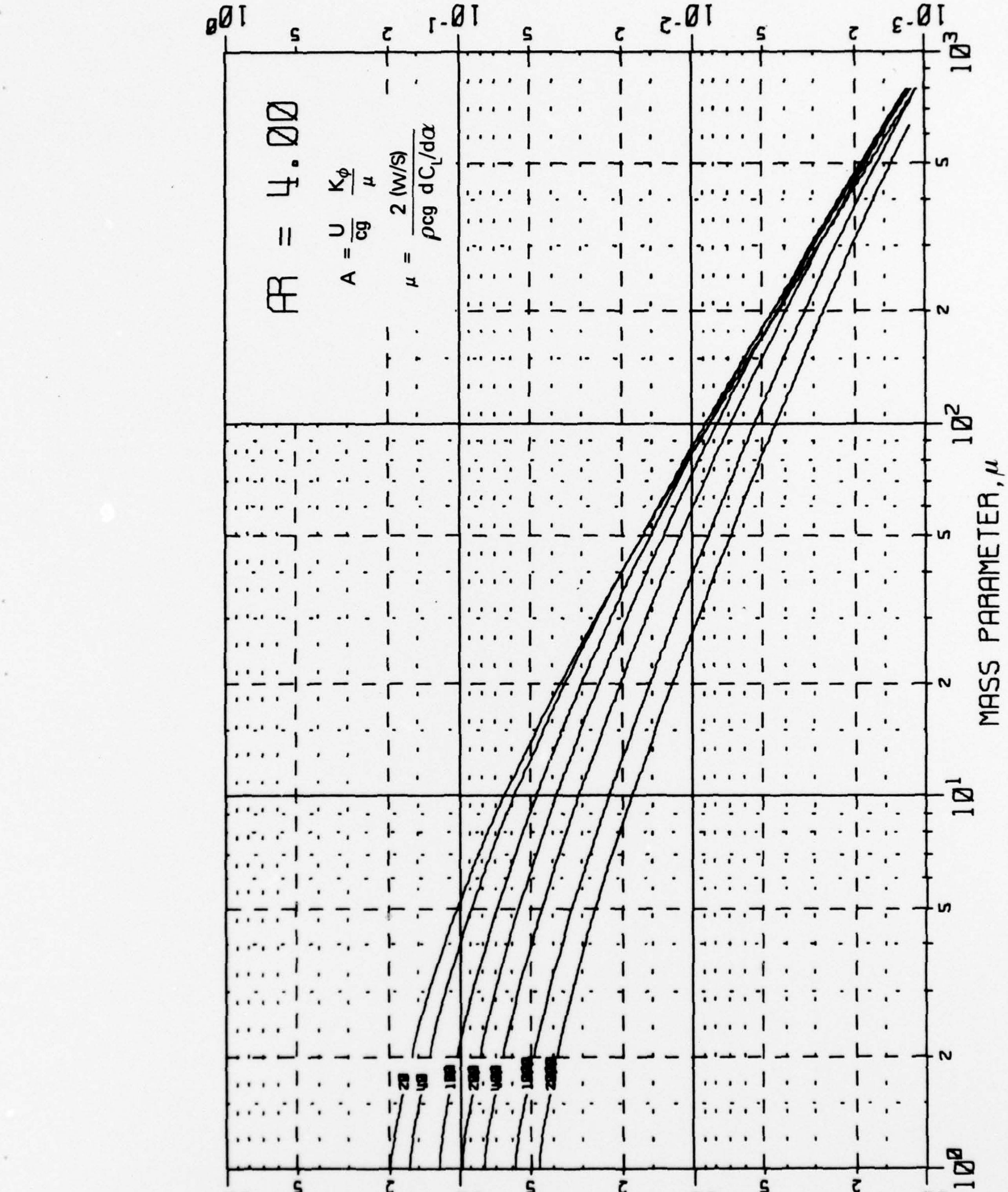

AR = 4.00
$A = \frac{U}{cg}\,\frac{K_\phi}{\mu}$
$\mu = \frac{2\,(W/S)}{\rho cg\; dC_L/d\alpha}$
K_ϕ / μ
MASS PARAMETER, μ
10^0
10^1
10^2
10^3
10^{-1}
10^{-2}
10^{-3}

FIG.6 (Cont)

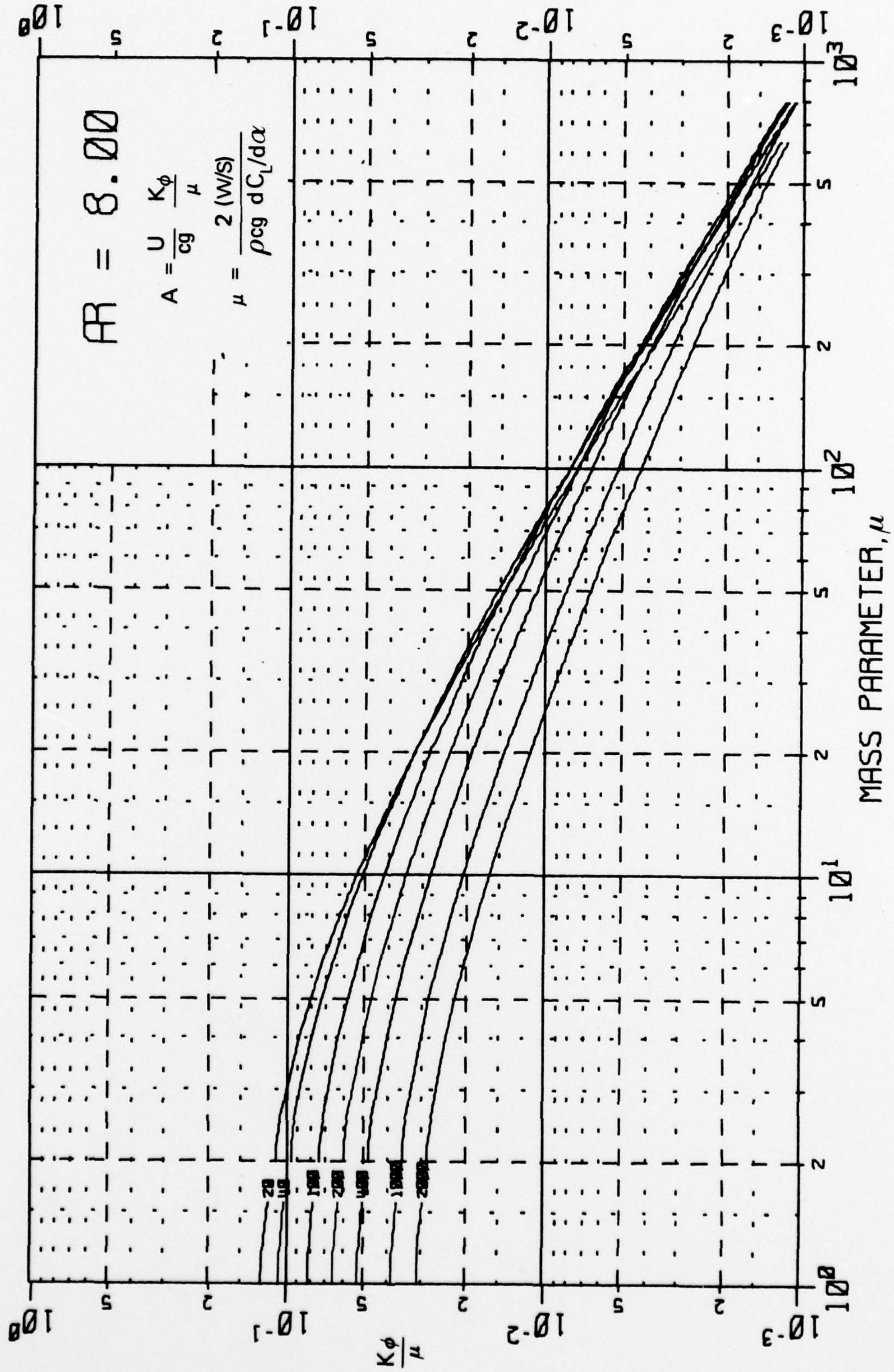
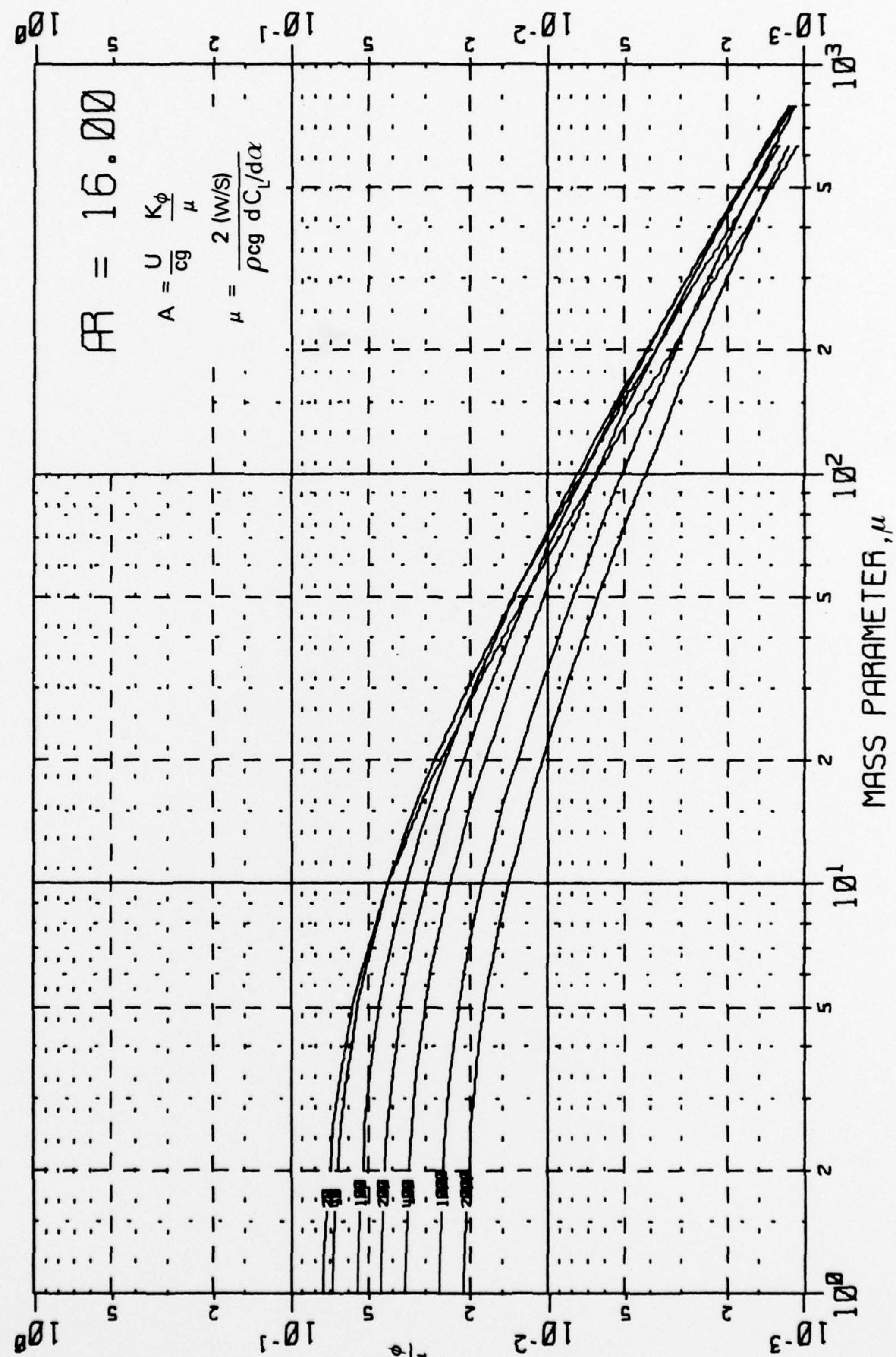

AR = 8.00
$A = \frac{U}{cg} \frac{K_\phi}{\mu}$
$\mu = \frac{2\,(W/S)}{\rho cg\, dC_L/d\alpha}$
MASS PARAMETER, μ
$\frac{K_\phi}{\mu}$
10^0
10^1
10^2
10^3
10^{-1}
10^{-2}
10^{-3}

FIG. 6 (Cont)



AR = 16.00
$A = \frac{U}{cg} \frac{K_\phi}{\mu}$
$\mu = \frac{2\,(W/S)}{\rho cg\; dC_L/d\alpha}$
MASS PARAMETER, μ
$\frac{K_\phi}{\mu}$
10^0
10^1
10^2
10^3
10^{-1}
10^{-2}
10^{-3}

FIG. 6 (Cont)

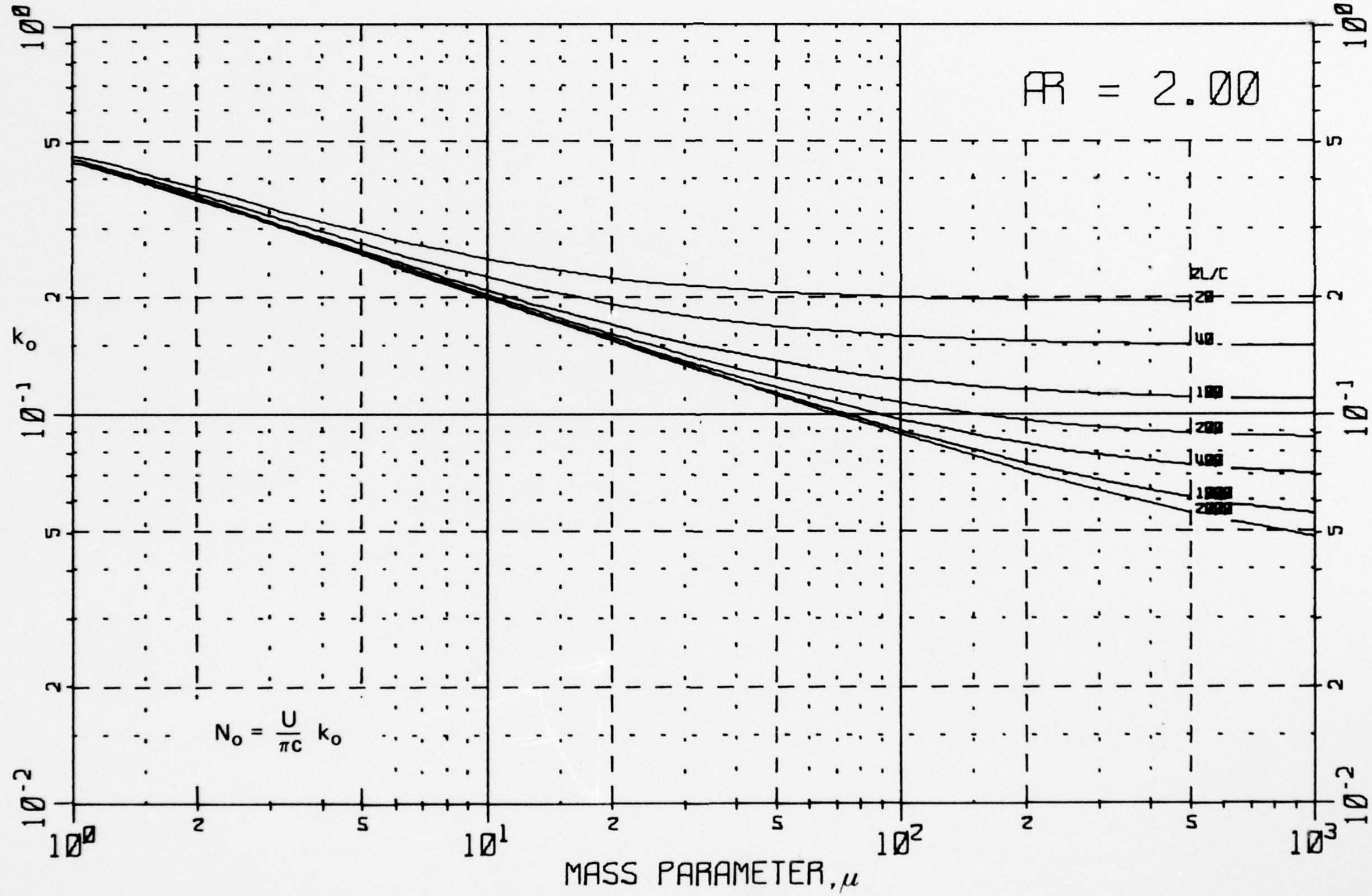


FIG. 7 CHARTS FOR THE DETERMINATION OF N_o FOR c.g. VERTICAL ACCELERATION

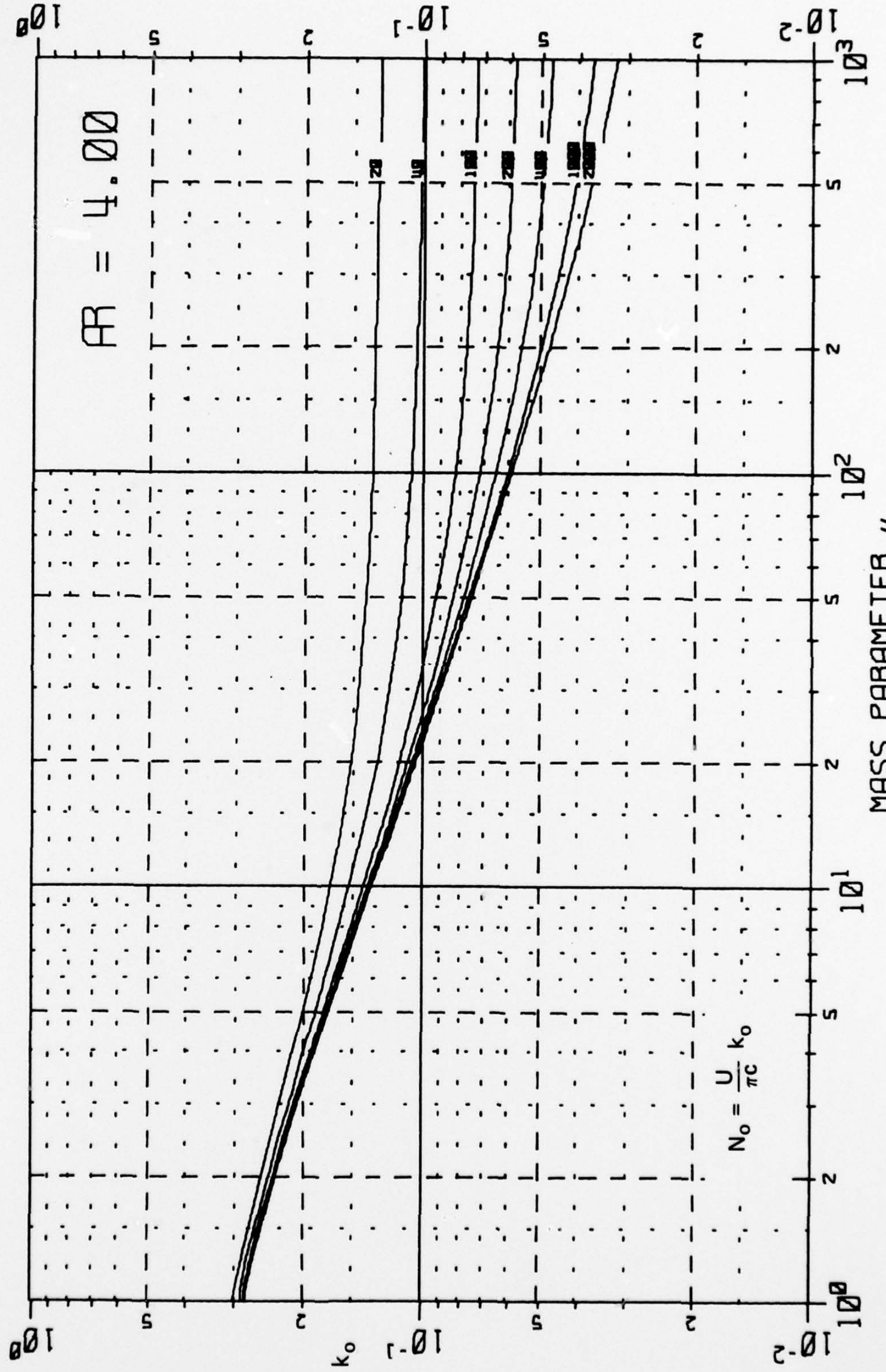


FIG. 7 (Cont)

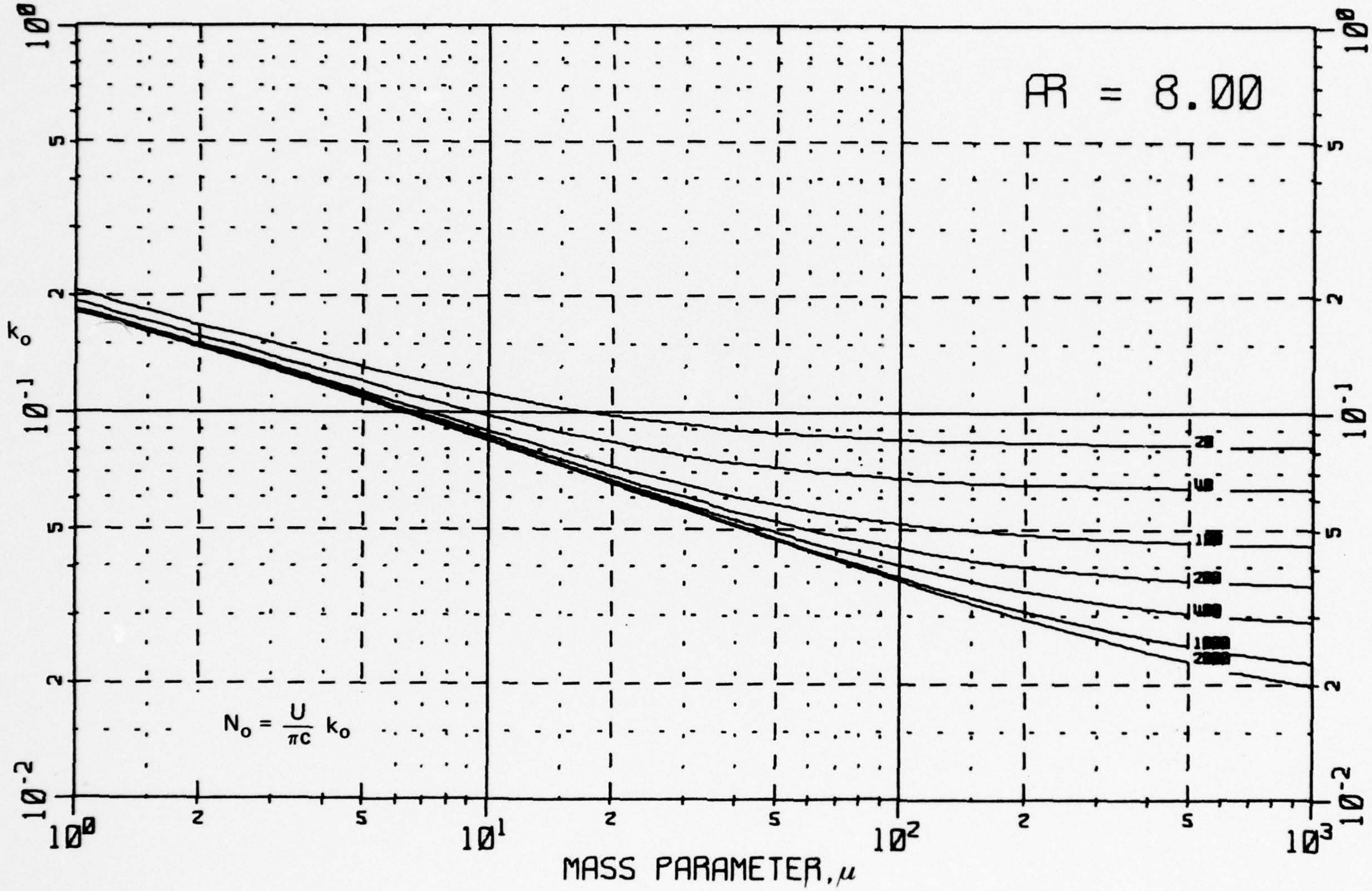

AR = 8.00
k_o
$N_o = \frac{U}{\pi c} k_o$
20
40
100
200
400
1000
2000
MASS PARAMETER, μ

FIG. 7 (Cont)

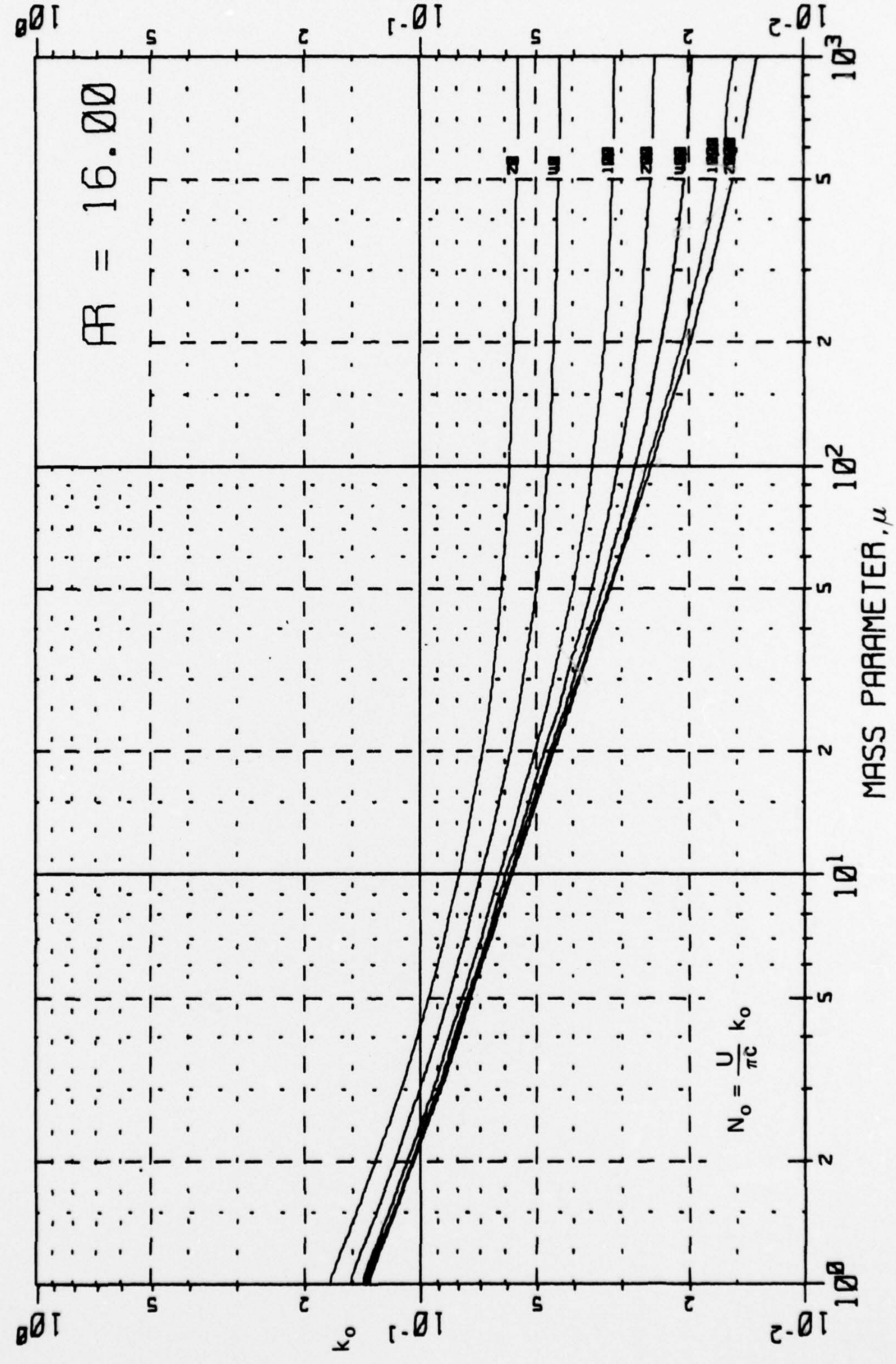
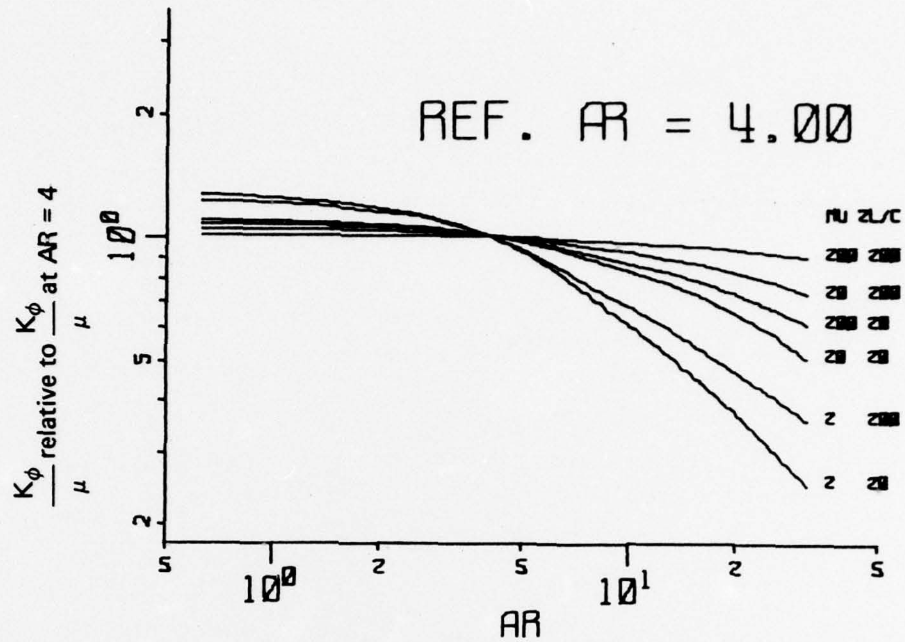

AR = 16.00
MASS PARAMETER, μ
k_o
$N_o = \frac{U}{\pi c} k_o$
10^0
10^1
10^2
10^3
10^{-1}
10^{-2}
20
30
100
200
300
1000
2000

FIG. 7 (Cont)



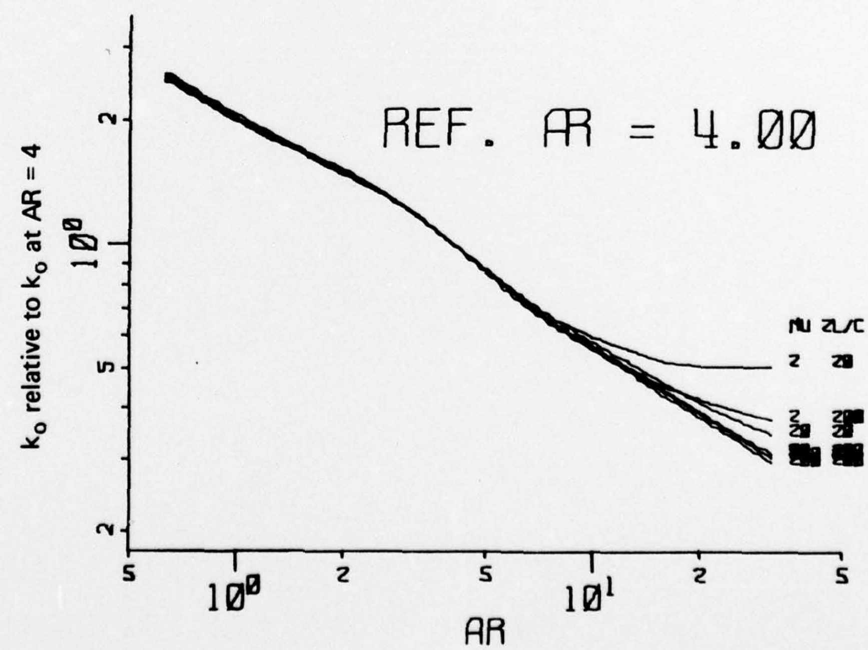


FIG. 8 EFFECT OF ASPECT RATIO ON PARAMETERS A AND N_o

DISTRIBUTION

AUSTRALIA

DEPARTMENT OF DEFENCE

	Copy No.
Central Office	
Chief Defence Scientist	1
Executive Controller, A.D.S.S.	2
Superintendent—Defence Science Administration	3
Controller—Service Laboratories and Trials Division	4
Controller—Military Studies and Operational Analysis Division	5
Controller—Programme Planning and Policy Division	6
Central Library	7
S.T.I.B.	8
J.I.O.	9
Aeronautical Research Laboratories	
Chief Superintendent	10
Superintendent—Structures Division	11
Structures Divisional File	12
D. J. Sherman (Author)	13
B. E. Anderson	14
F. H. Hooke	15
P. J. Howard	16
G. Long	17
A. O. Payne	18
C. K. Rider	19
M. R. Thomson	20
J. G. Sparrow	21
Library	22
Central Studies Establishment	
Library	23
Weapons Research Establishment	
Library	24

(14) ARL/STRUC NOTE-431

DOCUMENT CONTROL DATA

1. Security Grading/Release Limitation

   (a) Document Content: Unclassified

   (b) This page: Unclassified

2. Document Type/Number
   Structures Note 431

(11) 3. Document Date
   Aug[redacted]76

4. Title and Sub-Title (6) THE EFFECT OF SPANWISE GUST VARIATIONS ON THE TRANSFER FUNCTION OF AN AIRCRAFT MODEL WITH ONE DEGREE OF FREEDOM,

5. Personal Author (10) DOUGLAS JOHN/SHERMAN

(12) 31p.

6. Corporate Author(s) AERONAUTICAL RESEARCH LABORATORIES

7. *ABSTRACT*

Charts are derived for the determination of power spectrum parameters of A *and* N_0 *for the vertical acceleration, at an aircraft's centre of gravity, due to atmospheric turbulence. The method takes account of spanwise variations in the gust loading, and so overcomes the paradox of an infinite* N_0 *which is found with a 1-D gust model.*

8. Computer Program(s)—Titles and Language

008 650

9. Descriptors

   Flight Characteristics
   Turbulence
   Gust Loads
   Accelerometers
   Aircraft

11. Cosati Classifications
    1407, 2004, 2011, 0103, 1402

12. Task Reference (RD/P)

10. Library Distribution (Defence Group)

    Central Library
    A.R.D.U. Laverton
    S.T.I.B.
    J.I.O.
    A.R.L.
    C.S.E.
    W.R.E.
    S.T.S.O.

13. Sponsoring Agency Reference

14. Cost Code
    24 7010

15. Imprint MELBOURNE—AERONAUTICAL RESEARCH LABORATORIES, 1976